Evaluate
https://www.linkedin.com/groups/8313943/8313943-6430298795304722434
Σ(k = 1 to ∞) (1/2^{k}) tan (π/2^(k + 1)).
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Since $\cot\alpha - \tan\alpha = 2\cot 2\alpha$ then $\tan\alpha = \cot\alpha - 2\cot 2\alpha$ and for $\alpha = \frac{\pi}{2^{n+1}}$ we obtain
$\tan\frac{\pi}{2^{n+1}} = \cot\frac{\pi}{2^{n+1}} - 2\cot\frac{\pi}{2^n} n \in \mathbb{N}$.
Hence, $\sum_{k=1}^{n}\frac{1}{2^k}\tan\frac{\pi}{2^{k+1}} = \sum_{k=1}^{n}\left(\frac{1}{2^k}\cot\frac{\pi}{2^{k+1}} - \frac{1}{2^{k-1}}\cot\frac{\pi}{2^k}\right) =$
$\frac{1}{2^n}\cot\frac{\pi}{2^{n+1}} - \frac{1}{2^{1-1}}\cot\frac{\pi}{2^1} = \frac{1}{2^n}\cot\frac{\pi}{2^{n+1}}$ and, therefore,
$\sum_{n=1}^{\infty}\frac{1}{2^n}\tan\frac{\pi}{2^{n+1}} = \lim_{n\to\infty}\frac{1}{2^n}\cot\frac{\pi}{2^{n+1}} = \frac{2}{\pi}\lim_{n\to\infty}\frac{\frac{\pi}{2^{n+1}}}{\sin\frac{\pi}{2^{n+1}}} \cdot \lim_{n\to\infty}\cos\frac{\pi}{2^{n+1}} = \frac{2}{\pi}$.